

MATH 110: Old final exam

Justify all solutions fully and show your work.

- Write the (interesting) number 163 in the following numeral systems:
 - Base 2 (binary)
 - Base 6 (senary) *Remark: This numeral system is used by the Kanum- and Ndom-speaking peoples of New Guinea.*
 - Base 16 (hexadecimal)
- Recall that there are 23 students in our class.
 - Suppose that each person in our class lists the following 3 types of ice cream from favorite to least favorite (without ties): chocolate, strawberry, vanilla. Can we conclude that at least two students will have the same list of preferences?
 - If we instead ask each person in our class to rank 4 types of ice cream—suppose we add pistachio as an option—can we conclude that at least 2 students will have the same list of preferences?
- Use a tree diagram to find the prime factorization of 496.
- Consider a set X with 4 elements.
 - How many distinct functions $X \rightarrow X$ are there?
 - How many of the functions $X \rightarrow X$ are bijective?
 - A function $f : X \rightarrow X$ is an **involution** if $(f \circ f)(x) = x$. How many functions $X \rightarrow X$ are involutions?
- For each of following binary operations, (i) identify (and justify) whether it is commutative, and (ii) identify (and justify) whether it is associative.
 - (The averaging operation)** $(\mathbb{R}, *)$, where $a * b = \frac{1}{2}(a + b)$
 - (X, \star) , where X is any set (with $|X| > 1$) and $a \star b = a$
- In a few sentences, identify and explain some considerations when choosing a map projection to display geographical information. Give some examples of specific applications and projections appropriate for them.