

MATH 110: Final exam

September 8

Justify all solutions fully and show your work.

- (Converting between numerical bases)** Write the (interesting) number 137 in the following numeral systems:
 - Base 2 (binary)
 - Base 5 (quinary) *Remark: This numeral system is used by the Gumatj-speaking people of the Northern Territory, Australia.*
 - Base 16 (hexadecimal)
- (Prime factorization)** Use a tree diagram to find the prime factorization of 504.
- (Simple graphs with $|\mathcal{V}| = |\mathcal{E}| = 5$.)**
 - Draw all simple graphs with 5 (unlabeled) vertices and 5 edges. *Hint: There are 6 of them.*
 - Since the complete graph K_5 on 5 vertices has $\binom{5}{2} = 10$ edges, the dual of any graph with 5 vertices and 5 edges has 5 vertices and $10 - 5 = 5$ edges, hence it is one of the graphs you found in (a). For each graph you found in (a), identify which of the graphs you found in (a) is its dual.
- Consider a set X with 3 elements.
 - How many (distinct) functions $X \rightarrow X$ are there?
 - How many of the functions $X \rightarrow X$ are bijective?
 - How many of the functions $f : X \rightarrow X$ satisfy $f \circ f \circ f = \text{id}_X$ (that is, satisfy $(f \circ f \circ f)(x) = x$ for all x)?
 - A function $f : X \rightarrow X$ is **idempotent** if $f \circ f = f$. How many functions $X \rightarrow X$ are idempotent?
- For both of following binary operations,
 - identify (and justify) whether it is commutative, and
 - identify (and justify) whether it is associative.
 - (The difference operation)** (\mathbb{R}, \ominus) , where $a \ominus b = |a - b|$
 - (B, \circ) , where B is the set of bijections from $\{1, 2, 3\}$ to itself, and \circ is composition of functions.
- (Voting theory)** In a few sentences, explain several methods of voting (more precisely, processes for taking individual lists of preferences from every voter and producing a list of overall, i.e., social, preferences), and identify advantages and disadvantages of each.