

MATH 110: Class 01

Wednesday, August 17

For Thursday, August 18 (Algebra, Sets, Number sequences and pattern-finding):

Reading *A Quick Puzzle to Test Your Problem Solving*, David Leonhardt, New York Times, 2015 July 3.

Homework Submit the introductory survey.

Introduction

0. Course syllabus and class logistics.

- **Course description.**
- **Learning objectives.**
 - examine historical examples of mathematics,
 - identify mathematical thought processes,
 - relate logical relationships and computational tasks,
 - explain the connections between algebraic objects (roughly speaking, equations) and geometry (again roughly speaking, diagrams),
 - identify and demonstrate applications of mathematical thinking.
- **Learning resources.**
 - your peers, your instructor.
- **Graded components.**
 - participation
 - work outside class
 - project
 - final exam
- **Rules and expectations.**
 - Give your full attention; do not engage in unrelated (esp. electronic) activities.
 - Participate
 - Behave courteously to classmates.

1. What is mathematics? What is it for? Whom is it for?

2. Pólya's system. See *How to Solve It: A New Aspect of Mathematical Method*, George Pólya, 2nd ed.

- (a) Understand the problem
- (b) Devise a plan
- (c) Carry out the plan
- (d) Look back

Icebreaker

ACTIVITY (Exploring discrete dynamics: meet your classmates)

- (a) Introductions
- (b) (Silently) pick someone else in the classroom (—that you don't already know); we'll use this data to construct a discrete dynamical system.
- (c) What observations can we make about our system? About systems constructed this way in general?

Numbers and numeral systems

1. **What are numbers? Why bother with them?**
2. **Unary notation.** Tally marks.
3. **Sign-value notation.** A symbol represents a fixed value irrespective of position.

EXAMPLE (**Egyptian numerals**) (c. 3000 B.C.E.—by 1500 B.C.E. more sophisticated systems were in use).

	1
∩	10
ϣ	100
Ⲛ	1000
Ⲙ	10,000
ⲙ	100,000
ⲙⲙ	1,000,000

EXERCISE Write your birth year in Egyptian numerals.

EXAMPLE (**Roman numerals**) Similar to Egyptian numerals, but not a true sign-value system: The placement of a lower-value symbol before a higher-value one means that you subtract the lower value from the total, not add it.

- **What are the disadvantages of sign-value notations?**
4. **Positional notation (a.k.a. “place values”).** The value of a symbol depends on its position.
- **Review of decimal notation**
 - The decimal place value system.
 - Why decimal, i.e., why powers of ten?

EXAMPLE **Khipu (quipu)** Incan (and some pre-Incan) cultures used knotted fibers called *khipu* as data structures to record numerical information, including accounting records, in the decimal system. The full scope of their historical uses are unknown, but they were used for transmission of secret messages during an 18th C. rebellion against the Spanish Imperial government. (We’ll revisit the topic of encoding and decoding secret messages—cryptography—at the end of the course.) Much more can be said about khipu; further study of them would make a good topic for a course project.



- **Binary:** Base 2. Only two digits, 0 and 1. Binary is the native mathematical setting of nearly all computer science.
 - Binary is closely connected with Boolean logic, which we’ll see in Class 5.

EXAMPLE

$$(a) 1101_2 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 8 + 4 + 1 = 13_{10}$$

$$(b) 42_{10} = 32 + 8 + 2 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^0 = 10101_2.$$

EXERCISE.

(a) Write 101101_2 in decimal.

(b) Write your age (rounded down to the nearest whole number of years) in binary.

(c) Count from 1_2 to 10000_2 (in binary).

- **Hexadecimal:** Base 16; digits 0–9, A, B, C, D, E, F. Used widely in computer science.
- **Some other bases:** Base 10, together with the familiar symbols 0, 1, . . . , 8, 9—the Hindu-Arabic numeral system—is by far the most widely used (at least now), but many cultures use other systems.

EXAMPLE. Positional numbering systems adopted by some cultures (this list is *far* from exhaustive)

* Base 5 (quinary): Kanum, Nambu

* Base 6 (senary): Tonda

* Base 8 (octal): Northern Pame; also used in some computing applications

* Base 12 (duodecimal a.k.a. dozenal): Plateau languages of N. Nigeria; counting in base 12 in English language (dozen, gross, great gross)

* Base 20 (vigesimal): Classical Nahuatl (Aztec), Maya; poetic use in English (score)

EXAMPLE (**Babylonian cuneiform numerals**). Babylonian numbers are base 60 (sexagesimal). Sixty unrelated symbols would be hard to use, so the symbols for 0, . . . , 59 follow a pattern that uses a special symbol for multiples of 10.

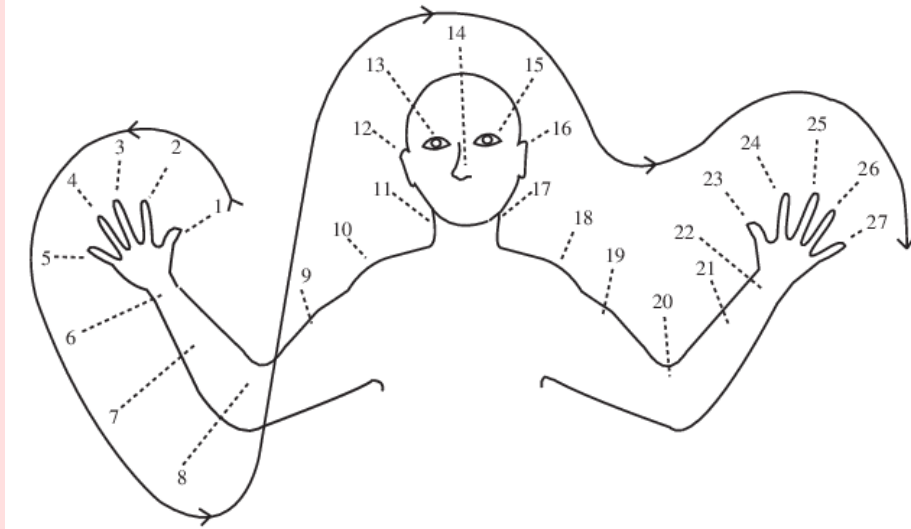
	1		11		21		31		41		51
	2		12		22		32		42		52
	3		13		23		33		43		53
	4		14		24		34		44		54
	5		15		25		35		45		55
	6		16		26		36		46		56
	7		17		27		37		47		57
	8		18		28		38		48		58
	9		19		29		39		49		59
	10		20		30		40		50		

When discussing Babylonian numbering now, we usually use decimal notation to express individual “digits” instead of using the cuneiform symbols, and separate the digits by colons. For example, for 2021 we would write 33 : 41.

We have some vestiges of the Babylonian counting system in our own measurement systems today; what are they?

- **Other systems**
 - **Another example**

EXAMPLE (Oksapmin body part enumeration) Both parts are used to count from 1 to 27.



ACTIVITY (Using numerical systems) Pick a number, ideally with several digits, e.g., your birth year.

(a) Write the number in:

- i. ternary (base 3)
- ii. hexadecimal
- iii. Babylonian cuneiform numerals
- iv. Roman numerals

(b) Record the number using khipu.