

CLASSIFICATION OF NONRIGID HARMONIC CURVATURE COMPONENTS OF PARABOLIC GEOMETRIES GIVEN BY BINARY FORMS

TRAVIS WILLSE

This table lists all of the nonrigid harmonic curvature components of parabolic geometries of type (G, P) with G simple given by binary forms on some bundle.

We list these in two groups: (1) The nonrigid harmonic curvature components of parabolic geometries whose marked Dynkin diagrams have a single uncrossed node, or equivalently, those for which $\mathfrak{g}_0^{ss} \cong \mathfrak{sl}(2, \mathbb{F})$. (2) Those for which there is more than one uncrossed node, so that $\mathfrak{g}_0^{ss} \cong \mathfrak{sl}(2, \mathbb{F}) \times \mathfrak{h}$ for some nontrivial semisimple Lie algebra \mathfrak{h} and which the restriction to \mathfrak{h} is trivial.

T.W.: FAKULTÄT FÜR MATHEMATIK, UNIVERSITÄT WIEN, OSKAR-MORGENSTERN-PLATZ 1, 1090 WIEN, AUSTRIA
Email address: travis.willse@univie.ac.at

TABLE 1. Curvature components for geometries with $\mathfrak{g}_0^{ss} \cong \mathfrak{sl}(2, \mathbb{F})$

degree	type	dim	geometry	word	curvature component
septic	G_2/P_2	5	G_2 contact	(12)	[7, -4]
sextic	B_3/P_{12}	8	causal	(21)	[1, -4, 6]
quintic	C_3/P_{13}	8		(13)	[-4, 5, -2]
	C_3/P_{23}	8		(23)	[5, -4, 1]
quartic	A_3/P_{12}	5	path	(21)	[0, -4, 4]
	A_3/P_{13}	5	Lagrangean contact	(13)	[-3, 4, -3]
	B_3/P_{12}	8	causal	(12)	[-4, 0, 4]
	C_2/P_2	3	conformal	(21)	[4, -5]
	C_3/P_{12}	8	contact path	(21)	[0, -5, 4]
	C_4/P_{124}	15	twistor space for C_4/P_2	(21)	[0, -5, 4, 0]
	G_2/P_1	5	(2, 3, 5) distribution	(12)	[-8, 4]
cubic	A_4/P_{124}	9		(21)	[-0, -4, 3, 1]
	A_5/P_{1245}	14	twistor space for A_5/P_{24}	(21)	[0, -4, 3, 0, 1]
	A_5/P_{1245}	14	twistor space for A_5/P_{24}	(45)	[1, 0, 3, -4, 1]
	C_2/P_1	3	contact projective	(12)	[-6, 3]
quadratic	A_3/P_{12}	5	path	(12)	[-4, 1, 2]
	A_4/P_{124}	9		(14)	[-3, 2, 2, 3]
	B_3/P_{13}	8	twistor space for (3, 6) distribution	(32)	[2, 2, -6]
	B_3/P_{23}	8	(another) twistor space for (3, 6) distribution	(32)	[2, 2, -6]
	C_3/P_{13}	8		(12)	[-5, 2, 1]
	D_4/P_{234}	11		(32)	[2, 0, -4, 2]
	D_4/P_{234}	11		(42)	[2, 0, 2, -4]
linear	A_2/P_1	2	projective	(12)	[-5, 1]
	A_3/P_{13}	5	Lagrangean contact	(12)	[-4, 1, 2]
	A_3/P_{13}	5	Lagrangean contact	(32)	[2, 1, -4]
	A_4/P_{124}	9		(12)	[-4, 1, 1, 1]
	A_5/P_{1234}	14	twistor space for almost Grassmannian	(21)	[0, -4, 3, 0, 1]
	C_3/P_{12}	8	contact path	(12)	[-5, 2, 1]
scalar	A_3/P_{12}	5	path	(23)	[4, -4, 0]
	A_5/P_{1235}	14	another twistor space for almost Grassmannian	(21)	[0, -4, 3, 0, 1]
	C_3/P_{23}	8	twistor space for generic (4, 7) distribution	(21)	[0, -5, 4]
	C_4/P_{123}	15	another twistor space for C_4/P_2	(21)	[0, -5, 4, 0]

The geometries appearing on the list with multiple irreducible harmonic curvature components are: A_3/P_{12} (3), A_3/P_{13} (3), A_4/P_{124} (3), A_5/P_{1245} , B_3/P_{12} , C_3/P_{12} , C_3/P_{13} , C_3/P_{23} , D_4/P_{234}

TABLE 2. Curvature components for geometries with $\mathfrak{g}_0^{\text{ss}} \cong \mathfrak{sl}(2, \mathbb{F}) \times \mathfrak{h}$

degree	type	dim	geometry	word	curvature component
quartic	A_3/P_2	4	conformal	(21)	$[0, -4, 4]$
	A_3/P_2	4	conformal	(23)	$[4, -4, 0]$
	C_3/P_2	7	generic (4, 7) distribution	(21)	$[0, -5, 4]$
	C_4/P_{24}	13		(21)	$[0, -5, 4, 0]$
	C_ℓ/P_{124}	$\ell \geq 5$	$8\ell - 17$	(21)	$[0, -5, 4, 0, \dots, 0]$
cubic	A_4/P_{13}	8		(34)	$[1, 3, -4, 0]$
linear	A_4/P_{23}	8	generic (4, 8) distribution	(21)	$[0, -4, 3, 1]$
	A_4/P_{23}	8	generic (4, 8) distribution	(34)	$[1, 3, -4, 0]$
	$A_\ell/P_{123, \ell-1}$	$\ell \geq 6$	$5\ell - 11$	(21)	$[0, -4, 3, 0, \dots, 0, 1]$

The geometries appearing on the list with multiple irreducible harmonic curvature components that are binary forms are A_3/P_2 and A_4/P_{23} .