# CR TRACTOR GEOMETRY: ALMOST CR-EINSTEIN STRUCTURES

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Abstract. We give a skeletal review of CR tractor geometry, just enough to state and frame some definitions and results related to almost CR-Einstein structures. We also give an annotated bibliography for the tractor geometry of CR structures.

## 1. Background

1.1. **The standard representation $\mathbb{T}$ of $SU(p+1, q+1)$.** Let $\mathbb{T} \cong \mathbb{C}^{p+1,q+1}$ denote the standard (defining) representation of the special unitary group $SU(p+1, q+1)$ of signature $(p+1, q+1)$ and denote a preserved Hermitian structure by $(h_{A\bar{B}}, \mathbb{J}^A{}_B)$. Fix an complex line $\ell$ isotropic with respect to $h$, and denote by $P$ the stabilizer of $\ell$ in $SU(p+1, q+1)$ of the induced action on $P\mathbb{T} \cong \mathbb{C}P^{n+1}$, $n := p+q$. Then, $P$ preserves the filtration

$$0 \subset \ell \subset \ell^\perp \subset \mathbb{T};$$

we denote

$$\mathbb{T}^{+1} := \ell, \quad \mathbb{T}^0 := \ell^\perp, \quad \mathbb{T}^{-1} := \mathbb{T}$$

and write $\mathbb{T}$ as the composition series

$$\mathbb{T} = (\mathbb{T}/\ell^\perp) \mathbin{+\!\!\!\!\supset} (\ell^\perp/\ell) \mathbin{+\!\!\!\!\supset} \ell.$$

A choice of isotropic line $m \in \mathbb{T}$ transverse to $\ell^\perp$ determines identifications $\mathbb{T}/\ell^\perp \cong m$ and $\ell^\perp/\ell \cong \ell^\perp \cap m^\perp$ and so a decomposition

$$\mathbb{T} = m \oplus (\ell^\perp \cap m^\perp) \oplus l.$$

With respect to a basis $(L, E_1, \ldots, E_{p+q}, M)$ that respects this splitting and satisfies $h_{A\bar{B}} L^A M^{\bar{B}} = 1$, the Hermitian metric $h$ has the form

$$h_{A\bar{B}} = \begin{pmatrix} & & 1 \\ & \mathbf{h} & \\ 1 & & \end{pmatrix}.$$

1.2. **CR tractor geometry.** Recall that the general theory of parabolic geometries canonically encodes any (integrable, Levi-nondegenerate, hypersurface-type) CR structure $(M, \mathbf{H}, \mathbf{J})$ as a Cartan geometry $(\mathcal{G} \to M, \omega)$ of type $(SU(p+1, q+1), P)$, where $(p, q)$ is the signature of Levi form.[1] [4, § 4.2.4] Here, $\mathcal{G} \to M$ is a principal

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[1]Strictly speaking, parabolic geometries of type $(SU(p+1, q+1), P)$ (satisfying the normality and regularity normalization conditions) correspond to CR structures equipped with a choice of $(n+2)$nd root of the canonical bundle $\bigwedge^{n+1}(H^{0,1})^\perp$, which we denote $\mathcal{E}(1,0)$. For any CR structure such a root always exists locally but need not exist globally; different choices do not play a substantive role in the material here. A type of parabolic geometries corresponding to CR

$P$-bundle and $\omega : T\mathcal{G} \mapsto \mathfrak{su}(p+1, q+1)$ is a Cartan connection; in particular, $\omega$ restricts to an isomorphism $\omega_u : T_u\mathcal{G} \overset{\cong}{\mapsto} \mathfrak{su}(p+1, q+1)$ of vector spaces at each $u \in \mathcal{G}$ and is appropriately equivariant with respect to the canonical right $P$-action on $\mathcal{G}$. We can view any representation $\mathbb{V}$ of $SU(p+1, q+1)$ and defined the associated **tractor bundle** [4, § 1.5.7]

$$\mathcal{V} := \mathcal{G} \times_P \mathbb{V}$$

over $M$; the typical fiber is isomorphic to $\mathbb{V}$. There, $\mathcal{G} \times_P \mathbb{V} := (\mathcal{G} \times P)/\sim$, where $(u, v) \sim (u \cdot p, p^{-1} \cdot v)$ for all $(u, v) \in \mathcal{G} \times \mathbb{V}$, $p \in P$. Then, $\omega$ induces a vector bundle connection $\nabla^{\mathcal{V}}$ on $\mathcal{V}$.

The tractor bundle associated to the standard representation $\mathbb{T}$ is the **standard tractor bundle** $\mathcal{T}$ (or $\mathcal{E}^A$), and the induced connection $\nabla^{\mathcal{T}}$ is the **(standard) (normal) tractor connection**. (Every (finite-dimensional) representation of $SU(p+1, q+1)$ is a subrepresentation of $\mathcal{T}^{\otimes a} \otimes (\mathcal{T}^*)^{\otimes b}$ for some $a, b$.) Since tractor bundles are associated $P$-bundles, the filtration $(\mathbb{T}^i)$ induces a filtration $(\mathcal{T}^i)$ of $\mathcal{T}$. We can canonically identify [8, § 3]

$$\mathcal{T}^{-1}/\mathcal{T}^0 \leftrightarrow \mathcal{E}(0,1), \qquad \mathcal{T}^0/\mathcal{T}^{+1} \leftrightarrow \mathcal{E}^\alpha(-1,0), \qquad \mathcal{T}^{+1} \leftrightarrow \mathcal{E}(-1,0),$$

giving the composition series

$$\mathcal{E}^A \cong \mathcal{E}(0,1) \mathbin{+\!\!\!\!\supset} \mathcal{E}^\alpha(-1,0) \mathbin{+\!\!\!\!\supset} \mathcal{E}(-1,0).$$

It is common in this setting to work (equivalently) with the dual tractor bundle $\mathcal{T}^*$ (or $\mathcal{E}_A$) associated to $\mathbb{T}^*$. Then, we can canonically identify

$$\mathcal{E}_A \cong \mathcal{E}(1,0) \mathbin{+\!\!\!\!\supset} \mathcal{E}_\alpha(1,0) \mathbin{+\!\!\!\!\supset} \mathcal{E}(0,-1).$$

A choice of pseudo-Hermitian structure $\theta$ (a nonvanishing section of $\mathbf{H}^\perp \subset T^*M$) determines a splitting

$$\mathcal{E}_A \overset{\theta}{=} \mathcal{E}(1,0) \oplus \mathcal{E}_\alpha(1,0) \oplus \mathcal{E}(0,-1);$$

with respect to this splitting, we can write a section $t_A \in \Gamma(\mathcal{T}) = \Gamma(\mathcal{E}_A)$ as

$$t_A \overset{\theta}{=} \begin{pmatrix} \rho \\ \tau_\alpha \\ \sigma \end{pmatrix} \in \Gamma \begin{pmatrix} \mathcal{E}(0,-1) \\ \mathcal{E}_\alpha(1,0) \\ \mathcal{E}(1,0) \end{pmatrix}.$$

The canonical projection $\mathbb{T} \to \mathbb{T}/\ell^\perp$ induces the projection $\Pi : \mathcal{T} \to \mathcal{E}(1,0)$ onto the bottom slot; by construction this projection does not depend on the choice $\theta$ of pseudo-Hermitian structure.

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structures without this additional discrete data is $(PSU(p+1, q+1), PP)$, where $PP$ is the image of $P$ under the quotient map $SU(p+1, q+1) \to PSU(p+1, q+1)$. Our main motivation for using $SU(p+1, q+1)$ rather than $PSU(p+1, q+1)$ is that the former admits faithful finite-dimensional representations, which play an essential role in the tractor calculus.

The corresponding tractor connection, which we also denote by $\nabla^{\mathcal{T}}$ or just $\nabla$, is given by

$$\nabla_\beta \begin{pmatrix} \rho \\ \tau_\alpha \\ \sigma \end{pmatrix} = \begin{pmatrix} \nabla_\beta \rho - \mathrm{P}_\beta{}^\gamma \tau_\gamma + T_\beta \sigma \\ \nabla_\beta \tau_\alpha + i A_{\alpha\beta}\sigma \\ \nabla_\beta \sigma - \tau_\beta \end{pmatrix}$$

$$\nabla_{\bar\beta} \begin{pmatrix} \rho \\ \tau_\alpha \\ \sigma \end{pmatrix} = \begin{pmatrix} \nabla_{\bar\beta} \rho - i A_{\bar\beta}{}^\alpha \tau_\alpha - T_{\bar\beta}\sigma \\ \nabla_{\bar\beta} \tau_\alpha + \mathbf{h}_{\alpha\bar\beta}\rho + \mathrm{P}_{\alpha\bar\beta}\sigma \\ \nabla_{\bar\beta}\sigma \end{pmatrix}$$

$$\nabla_0 \begin{pmatrix} \rho \\ \tau_\alpha \\ \sigma \end{pmatrix} = \begin{pmatrix} \nabla_0 \rho + \frac{i}{n+2}\mathrm{P}\rho + 2iT^\alpha \tau_\alpha + iS\sigma \\ \nabla_0 \tau_\alpha - i\mathrm{P}_\alpha{}^\gamma \tau_\gamma + \frac{i}{n+2}\mathrm{P}\tau_\alpha + 2iT_\alpha\sigma \\ \nabla_0\sigma + \frac{i}{n+2}\mathrm{P}\sigma - i\rho \end{pmatrix},$$

where on the right-hand-side $\nabla$ denotes the connection on the respective weighted bundles induced by the Tanaka-Webster connection corresponding to $\theta$,[2] where

$$\mathrm{P}_{\alpha\bar\beta} = \frac{1}{n+2}\left(R_{\alpha\bar\beta} - \frac{1}{2(n+1)} R\mathbf{h}_{\alpha\bar\beta}\right)$$
$$T_\alpha = \frac{1}{n+2}(\nabla_\alpha \mathrm{P} - i\nabla^\beta A_{\alpha\beta})$$
$$S = -\frac{1}{n}(\nabla^\alpha T_\alpha + \nabla^{\bar\alpha} T_{\bar\alpha} + \mathrm{P}_{\alpha\bar\beta}\mathrm{P}^{\alpha\bar\beta} - A_{\alpha\beta}A^{\alpha\beta}),$$

and where $\mathrm{P} := \mathrm{P}_\gamma{}^\gamma$.

## 2. Almost CR-Einstein structures

By analogy with the conformal setting [1], and following [2, § 4.14], we define an **almost CR-Einstein structure** to be a section $I_A \in \Gamma(\mathcal{T}^*)$ parallel with respect to the tractor connection $\nabla$.[3] If $I_A = \begin{pmatrix}\rho & \tau_\alpha & \sigma\end{pmatrix}^\perp$ is a parallel tractor, then expanding the bottom slot of $\nabla_\beta I_A = 0$ gives that $\tau_\alpha = \nabla_\alpha\sigma$, and then substituting and expanding the middle slot of $\nabla_{\bar\beta}\sigma$ gives

$$\nabla_{\bar\beta}\nabla_\alpha\sigma + \mathrm{P}_{\alpha\bar\beta}\sigma + \mathbf{h}_{\alpha\bar\beta}\rho = 0, \tag{1}$$

so that[4]

$$I_A \overset{\theta}{=} \begin{pmatrix} -\frac{1}{n}(\nabla^\beta\nabla_\beta\sigma + \mathrm{P}\sigma) \\ \nabla_\alpha\sigma \\ \sigma \end{pmatrix}.$$

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[2]The expressions on the right-hand side of formula for the derivative in the 0-direction simplify some when written in terms of the Weyl connection $\nabla^W$ that coincides with $\nabla$ in contact directions but satisfies $\nabla^W_0\sigma = \nabla_0\sigma + \frac{i}{n+2}\mathrm{P}\sigma$ and $\nabla^W_0\tau_\alpha = \nabla_0\tau_\alpha - i\mathrm{P}_\alpha{}^\beta\tau_\beta$. See [2, § 3.4.1].

[3]This is more than an analogy: Recall that the Fefferman construction canonically assigns to $(M, \mathbf{H}, \mathbf{J})$ a circle bundle $\mathcal{F} \to M$ equipped with a canonical **Fefferman conformal structure** $\mathbf{c}$. In particular, we can identify the standard tractor bundle of $(\mathcal{F}, \mathbf{c})$, and the normal CR and conformal tractor connections are related in a way that the pullback of a parallel section $t$ is a parallel conformal standard tractor. These latter correspond with **almost Einstein scales**, establishing a bijective correspondence between almost CR-Einstein structures of a CR structure and almost Einstein scales of the induced Fefferman conformal structure.

[4]The map $\Gamma(\mathcal{E}(1,0)) \to \Gamma(\mathcal{T})$ on sections that maps $\sigma$ to the section defined by the right-hand side does not depend on the choice of pseudo-Hermitian structure $\theta$ and is called the **first BGG splitting operator** associated to $\mathbb{T}$.

Now, the middle slot of $\nabla_\beta I_A = 0$ and the bottom slot of $\nabla_{\bar\beta} I_A = 0$ give that $\Pi(I) = \sigma$ satisfies the invariant system

$$\nabla_\alpha \nabla_\beta \sigma + i\sigma A_{\alpha\beta} = 0, \tag{2}$$

$$\nabla_{\bar\beta}\sigma = 0. \tag{3}$$

*Remark.* Since this system is invariant, so are the respective differential operators $\Theta^{(2)} : \Gamma(\mathcal{E}(1,0)) \to \Gamma(\bigodot^2 \mathbf{H}^*(1,0))$ (second order) and $\Theta^{(1)} : \Gamma(\mathcal{E}(1,0)) \to \Gamma(\mathbf{H}(1,0))$ (first order) defined by mapping $\sigma$ to the respective right-hand sides of (2), (3). Together these two maps comprise the **first BGG operator** for CR geometry associated to the standard representation $\mathbb{T}$.

The formula for $I_A$ in terms of $\sigma$ gives that $I_x$ depends only on the 2-jet of $\sigma$ at $x$, so if we denote $\Sigma := \{\sigma \neq 0\}$, then $M - \Sigma$ is an open, dense subset of $M$. Now, $\sigma\bar\sigma \in \Gamma(\mathcal{E}(1,1))$ is a scale on $M - \Sigma$ and determines a Hermitian metric $(\sigma\bar\sigma)^{-1}\mathbf{h}_{\alpha\bar\beta}$. If $\sigma$ vanishes nowhere (for example, taking $\sigma|_{M-\Sigma}$), we call $I$ a CR-Einstein structure and $\sigma$ a CR-Einstein scale; henceforth we work in this setting.

Using the corresponding pseudo-Hermitian connection $\nabla$, we have $\nabla_\alpha(\sigma\bar\sigma) = 0$, and then (3) implies $\nabla_\alpha\sigma = 0$. If $\theta$ is the corresponding pseudo-Hermitian form, then

$$I_A \overset{\theta}{=} \begin{pmatrix} -\frac{1}{n}\mathrm{P}\sigma \\ 0 \\ \sigma \end{pmatrix}. \tag{4}$$

Moreover, in this scale:

- Simplifying and rearranging (1) gives $\mathrm{P}_{\alpha\bar\beta} = (-\sigma^{-1}\rho)\mathbf{h}_{\alpha\bar\beta}$. Since $\mathrm{P}_{\alpha\bar\beta}$ is a trace adjustment of the Webster-Ricci tensor $R$ of $\theta$, this equation implies that $R$ has zero tracefree part, that is $\theta$ (equivalently, $\mathbf{h}_{\alpha\bar\beta}$) is *pseudo-Einstein* in the sense of [9].
- Simplifying (2), that is, $\Theta^{(2)}(\sigma) = 0$, gives $A_{\alpha\beta} = 0$, which is equivalent to the Reeb field $T$ corresponding to $\theta$ being an infinitesimal symmetry of the CR structure $(\mathbf{H}, \mathbf{J})$.

Pseudo-Hermitian structures that are pseudo-Einstein and whose corresponding Reeb field is an infinitesimal symmetry are called *transversely symmetric pseudo-Einstein structures (TSPEs)* [10].

These deductions can be reversed:

**Proposition 1.** [2, Proposition 4.14] *A contact form on a CR manifold* $(M, \mathbf{H}, \mathbf{J})$ *determines a TSPE structure iff the corresponding scale* $\sigma$ *is a CR-Einstein scale.*

*Proof.* It remains to prove the forward direction: Assume that for the contact form $\theta$ the Rho tensor $\mathrm{P}_{\alpha\bar\beta}$ is a multiple of the Hermitian form $\mathbf{h}_{\alpha\bar\beta}$. Now, suppose for $\sigma \in \Gamma(-1,0)$ that $\sigma\bar\sigma$ is the CR scale corresponding to $\theta$. The fact that the power $\sigma^{n+2} \in \Gamma(\mathcal{E}(-n-2,0)) = \Gamma(\bigwedge^{n+1}(H^{0,1})^\perp)$ is a volume form normalized with respect to $\theta$ determines $\sigma$ up to a phase. Now, [9, Theorem 4.2] shows that we may choose the phase so that $d(\sigma^{n+2}) = 0$, so $\nabla_{\bar\beta}\sigma = 0$.

Computing in the scale $\sigma\bar\sigma$ gives $\nabla_\beta\sigma = 0$, so $A_{\alpha\beta} = 0$ implies that $\sigma$ is a solution to (2)-(3). Next, the covariant commutator of $f \in \Gamma(\mathcal{E}(w, w'))$ is [8, (2.4)]

$$[\nabla_\alpha, \nabla_{\bar\beta}]f = (w - w')\left(\mathrm{P}_{\alpha\bar\beta} + \frac{1}{n+2}\mathrm{P}\mathbf{h}_{\alpha\bar\beta}\right)f - i\mathbf{h}_{\alpha\bar\beta}\nabla_0 f.$$

Specializing to $f = \sigma$ and rearranging gives $i\nabla_0\sigma = \frac{2(n+1)}{n(n+2)}\mathrm{P}\sigma$, and then differentiating gives $i\nabla_\alpha\nabla_0\sigma = \frac{2(n+1)}{n(n+2)}\sigma\nabla_\alpha\mathrm{P}$. Now, [8, (2.4)] implies that $\nabla_0$ and $\nabla_\alpha$ commute on densities, so $\nabla_\alpha\nabla_0\sigma = \nabla_0\nabla_\alpha\sigma = 0$ and thus $\nabla_\beta\mathrm{P} = 0$. An analogous argument gives $\nabla_{\bar\beta}\mathrm{P} = 0$, and so P is a constant. Putting this altogether gives that the quantity $I_A$ defined by (4) is annihilated by $\nabla_\beta, \nabla_{\bar\beta}, \nabla_0$. Thus, $\nabla I = 0$ and, since $\sigma$ vanishes nowhere, this defines a CR-Einstein scale on $M$. □

## Annotated bibliography


[1] T. Bailey, M. Eastwood, A.R. Gover, Thomas's structure bundle for conformal, projective and related structures, *Rocky Mountain J. Math.* **24** (1994), 1191–1217. https://projecteuclid.org/euclid.rmjm/1181072333 doi:10.1216/rmjm/1181072333
This article rediscovered tractor geometry via prolongation of the conformally invariant (almost) Einstein equation, the conformal analogue of equation defining almost CR-Einstein structures. (Thomas' original article is perhaps difficult for the typical modern reader, since his work predates the notion of vector bundle.) The article makes no explicit reference to CR geometry.

[2] A.R. Gover, A. Čap, CR-tractors and the Fefferman Space, *Indiana U. Math. J.* **57** (2008), 2519–2570. arXiv:math/0611938 doi:10.1512/iumj.2008.57.3359
This article describes using the framework of tractor geometry the classifical Fefferman construction and in particular establishes close relationships between the tractor geometries of the underlying CR structure and the induced conformal structure. As applications, it describes a decomposition of the infinitesimal symmetry algebra of a Fefferman conformal structure in terms of CR data, relates the CR and conformal tractor-$D$ and double-$D$ operators, and relates almost CR-Einstein and almost Einstein structures.

[3] A.R. Gover, A. Čap, Tractor calculi for parabolic geometries, *Trans. Amer. Math. Soc.* **354** (2002), 1511–1548. http://www.ams.org/journals/tran/2002-354-04/S0002-9947-01-02909-9/S0002-9947-01-02909-9.pdf doi:10.1090/S0002-9947-01-02909-9
This article introduces a general theory of tractor calculus for parabolic geometries. Most of the results specialize in the CR setting to those of [8], but § 3 generalizes the treatment of fundamental -operators substantially, even in CR setting. NB § 4 applies only to irreducible (|1|-graded) parabolic geometries and so does not apply to CR geometry.

[4] A. Čap, J. Slovák, *Parabolic Geometries I: Background and General Theory*, Math. Surv. and Monographs **154**, Amer. Math. Soc. (2009).
This panoramic text is the standard reference for the general theory of parabolic geometries. Material that specifically treats CR structures includes: § 1.6.6 (the CR sphere as a homogeneous space), Example 3.1.2(3) (the |2|-grading associated to CR geometry), Example 3.1.7 (interpretation of infintesimal flag structures and regularity in CR geometry), §§ 4.2.1-2 (generalities of parabolic contact structures, of CR structures are the most important example), § 4.2.4 (partially integrable almost CR structures as parabolic geometries), §§ 4.3.8-10 (codimension-2 CR structures on 6-manifolds), Example 4.5.1 and Corollary 4.5.2 (the classical Fefferman construction as a special case of the generalized Fefferman construction), § 4.5.5 (CR structures appearing in a twistorial construction for quaternionic contact structures), and § 5.2.17 (preferred connections associated to partially integral almost CR-structures). Aspects of tractor geometry are covered in §§ 1.5.7-12 (some generalities), §§ 3.1.21-22 (abstract tractor bundles and tractor descriptions of parabolic geometries), and § 5.1.10 and surrounding material (tractor connections for parabolic geometries).

[5] S. Curry, A.R. Gover, CR embedded submanifolds of CR manifolds. *Mem. Amer. Math. Soc.* (To appear.) arXiv:1511.07928
This monograph (essentially the first author's Ph.D. thesis) develops a general framework and many basic results for the relationship between the tractor geometries of a CR structure and another CR structure therein. Of special interest are the construction of invariants of CR embedded submanifolds (§ 7) and construction of tractor CR analogues of the Gauss-Codacci-Ricci equations and a CR Bonnet theorem (§ 8).

[6] C. Fefferman, Monge-Ampère equations, the Bergman kernel, and geometry of pseudoconvex domains, *Ann. Math.* **103** (1976), 395–416.

https://www.jstor.org/stable/1970945 doi:10.2307/1970945 Correction: *Ann. Math.* **104** (1976), 393–394. https://www.jstor.org/stable/1970961 doi:10.2307/1970961
This article introduces the classical Fefferman construction for embedded CR structures. It predates the development of CR tractor calculus by about two decades, but the material here was substantially influential in the development of modern tractor calculus.

[7] A.R. Gover Aspects of parabolic invariant theory. *Rend. Circ. Mat. Palermo Serie II, Suppl.* **59** (1999), 22–47. https://dml.cz/bitstream/handle/10338.dmlcz/701786/WSGP_18-1998-1_4.pdf 10.1.1.30.2300
This article, which is a companion to a lecture series given at Srni Winter School in 1998, gives in § 5 (joint with C.R. Graham) a first outline for CR tractor calculus, including a dedicated treatment of the tractor calculus on the CR sphere, and gives a survey of some (then) in-progress applications, including constructions of some basic differential operators and invariant powers of the CR sub-Laplacian operator of Jerison-Lee.

[8] A.R. Gover, C.R. Graham, CR Invariant Powers of the sub-Laplacian, *J. Reine Agnew. Math.* **583** (2005), 1–27. arXiv:math/0301092 doi:10.1515/crll.2005.2005.583.1
This article constructs in detail the CR tractor calculus and uses it to define various CR invariant differential operators. This includes the CR invariant powers $\mathcal{P}_{w,w'}$ of the sub-Laplacian $\Delta_b := (\nabla^\alpha \nabla_\alpha + \nabla^{\bar\alpha} \nabla_{\bar\alpha})$, that is, CR-invariant operators whose principal part agrees with that of $\Delta_b^k$. It also relates the previous constructions ot the Fefferman conformal structure.

[9] J.M. Lee, Pseudo-Einstein structures on CR manifolds, *Amer. J. Math.* **110** (1988), 157–178. https://www.jstor.org/stable/2374543 doi:10.2307/2374543
This article significantly predates the development of CR tractor geometry but anticipates one of its consequences by defining and analyzing pseudo-Einstein structures on CR manifolds.

[10] F. Leitner, On transversally symmetric pseudo-Einstein and Fefferman-Einstein spaces, *Math. Z.* **256** (2007), 443–459. https://link.springer.com/article/10.1007/s00209-007-0121-8 arXiv:math/0502287 (title differs from published version) doi:10.1007/s00209-007-0121-8
Motivated by CR tractor geometry, this article treats transversally symmetric pseudo-Einstein structures, which correspond to CR-Einstein scales (parallel standard tractors whose projecting parts have empty zero locus), and relates them to Kähler-Einstein geometry.